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and cuts the κ tangent in

$$x=\frac{\kappa+\lambda}{\lambda-\kappa};\ y=\frac{\kappa\lambda-1}{\lambda-\kappa}.$$

The line joining this to the cut of tangents at $(\mu,\ \nu)$ has the slope

$$\frac{\kappa(\lambda+\mu+\nu+\lambda\mu\nu)-(1+\lambda\mu+\mu\nu+\nu\lambda)}{2(\kappa+\lambda\mu\nu)}$$

which is symmetrical in $(\lambda,\ \mu,\ \nu)$. The three lines given by interchanges of λ, μ, ν are therefore parallel.

CALCULUS.

312. Proposed by C. N. SCHMALL, New York City.

Given $y^3-3y+x=0$, prove by Maclaurin's theorem, that

$$y=\frac{x}{3}+\frac{x^3}{3^4}+\frac{x^5}{3^6}+\text{etc.}$$

I. Solution by H. PRIME.

Put $u=y^3-3y+x=0$. Then $\partial u/\partial x=1$, $\partial u/\partial y=3y^2-3$; hence, $dy/dx=\frac{1}{3}(1-y^2)=(1+y^2+y^4+y^6+\text{etc.})/3$.

$$d^2y/dx^2=(2y+4y^3+6y^5+\text{etc.})(1+y^2+y^4+\text{etc.})/3^2$$
$$=(2y+6y^3+12y^5+\text{etc.})/3^2.$$

$$d^3y/dx^3=(2+18y^2+60y^4+\text{etc.})(1+y^2+y^4+\text{etc.})/3^3$$
$$=(2+20y^2+80y^4+\text{etc.})/3^3.$$

$$d^4y/dx^4=(40y+320y^3+\text{etc.})(1+y^2+y^4+\text{etc.})/3^4$$
$$=(40y+360y^3+\text{etc.})/3^4.$$

$$d^5y/dx^5=(40+1080y^2+\text{etc.})(1+y^2+y^4+\text{etc.})/3^5$$
$$=(40+1120y+\text{etc.})/3^5;\ \text{etc.}$$

When $x=0$, $(y)=0$. Hence

$(dy/dx)=\frac{1}{3}$, $(d^2y/dx^2)=0$, $(d^3y/dx^3)=2/3^3$, $(d^4y/dx^4)=0$, $(d^5y/dx^5)=40/3^5$, etc. By Maclaurin's formula, $y=x/3+x^3/3^4+x^5/3^6+\text{etc.}$

II. Solution by A. M. HARDING, University of Arkansas, and JAMES A. BULLARD, Worcester Polytechnic Institute.

Let $f(x,\ y)\equiv y^3-3y+x=0$. Then

$$\frac{dy}{dx}=-\frac{\partial f/\partial x}{\partial f/\partial y}=-\frac{1}{3y^2-3}=f'(x,\ y);\ \frac{d^2y}{dx^2}=\frac{-6y}{(3y^2-3)^3}=f''(x,\ y);$$

$$\frac{d^3y}{dx^3}=\frac{-6(15y^2+3)}{(3y^2-3)^5}=f'''(x,\ y);\ \frac{d^4y}{dx^4}=\frac{-24.45y(2y^2+1)}{(3y^2-3)^7}=f^{\mathrm{iv}}(x,\ y);$$

$$\frac{d^5y}{dx^5}=\frac{-120.9(56y^4+57y^2+3)}{(3y^2-3)^9}=f^{\mathrm{v}}(x,\ y);\ \ldots$$

Substituting the values of these derivatives for $x=0$, *i. e.* when $y=0$, in Maclaurin's formula, we have

$$y=f(0)+f'(0)x+\frac{f''(0)}{2!}x^2+\frac{f'''(0)}{3!}x^3+\frac{f^{\mathrm{iv}}(0)}{4!}x^4+\frac{f^{\mathrm{v}}(0)}{5!}x^5+\ldots$$

$$=\frac{x}{3}+\frac{x^3}{3^4}+\frac{x^6}{3^6}+\ldots$$

NOTE. When $x=0$, $y=0$, $\sqrt{3}$, or $-\sqrt{3}$. Then there are two other series for y obtained by substituting $y=\sqrt{3}$ and $y=-\sqrt{3}$ in the above derivations.

Solved similarly by V. M. Spunar, S. G. Barton, M. E. Graber, and C. N. Schmall.

313. Proposed by M. E. GRABER, A. M., Heidelberg University, Tiffin, Ohio.

Evaluate the definite integral $\int_0^\infty (e^{-2ax^2}+e^{2ax^2})\,dx$.

Solution by C. N. SCHMALL, New York City.

$$\int_0^\infty (e^{-2ax^2}+e^{2ax^2})\,dx=\int_0^\infty e^{-2ax^2}dx+\int_0^\infty e^{2ax^2}dx.$$

The first of these two integrals can be evaluated by meams of the familiar result $\int_0^\infty e^{-x^2}dx=\frac{1}{2}\sqrt{\pi}$ (see Williamson's *Integral Calculus*).

Putting $x\sqrt{(2a)}$ for x, we have

$$\int_0^\infty e^{-2ax^2}dx=\tfrac{1}{2}\sqrt{\frac{\pi}{2a}}.$$